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Formulation Of The Problem Of Instability Of Streaming Walters' Fluids In A Porous Medium

Dr. Lekh Raj

Assistant Professor, Punjabi university neighbourhood campus,
Karandi, Distt. Mansa (Punjab)
India
Corresponding Author e-mail: lekhraj2007@rediffmail.com

Abstract: - The instability of the plane interface separating the two uniforms, superposed, electrically conducting and counter - streaming elastico-viscous fluids through a porous medium is examined for visco-elastic polymer solutions in the presence of a horizontal magnetic field and also in the presence and absence of surface tension. These solutions are known as Walters' (model B μ) fluiare approximatedeology is approximated by the Walters' (model B μ) constitutive relations, proposed by Walters (1962). In the absence of surface tension, the perturbations transverse to the direction of streaming are found to be unaffected by the presence of streaming if perturbations in the direction of streaming are ignored, whereas for perturbations in all other directions, there exists the instability for a certain wave number range. The magnetic field and surface tension are able to suppress this Kelvin-Helmholtz instability for small wave length perturbations, and the medium porosity and the visco-elasticity reduce the stability range given in terms of a difference in streaming velocities and the Alfven velocity.

Key Words: - *Kelvin-Helmholtz (KH), Porous Medium, Kinematic Viscosities, Two Superposed Streaming Fluids, Walters' (model B μ) fluids.*

1. Introduction

The instability of the plane interface separating the two superposed semi - infinite fluids flowing with different velocities has been considered by Helmholtz (1868) and Kelvin (1910) and a review of this Kelvin - Helmholtz instability, under varying assumptions of hydrodynamics and hydro-magnets, has been given by Chandrashekhar (1961). Helmholtz stated that every perfect geometrically sharp edge by which a fluid flows out tear it as under and establish a surface of separation; however, slowly the rest of the fluid may more. A good review of the interface between two fluids in relative motion has been given by Gerwin (1968). Alterman (1961) has studied the effect of surface tension to the Kelvin-Helmholtz instability of two rotating fluids. Reid (1961) studied the effect of surface tension and viscosity on the stability of two superposed fluids. Bellman and Pennington (1954) further investigated in detail illustrating the combined effects of viscosity and surface tension. The medium has been assumed to be non - porous in the above studies.

Generally, the magnetic field has a stabilizing effect on the instability, but there are a few exceptions also. For example, Kent (1966) has studied the effect of a horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of the magnetic field the system is known to be stable. In stellar atmospheres and interiors, the magnetic field may be (and quite often is) variable and may altogether alter the nature of the instability. Coriolis force also plays an important role in the stability of the system. In all the above studies the fluid has been assumed to be Newtonian.

The theory of fluids with nonlinear constitutive equations, so called non-Newtonian fluids started by Reiner (1945) for compressible fluids, Rivlin (1948) for incompressible materials and broadened and elaborated by Ericksen (1953), Truesdell and Noll (1965) and many others. Polymer solutions and polymers melt, which provide the most common examples of Non-Newtonian fluids, are usually quite viscous; it is important to note that observable non-Newtonian effects occur at a fairly low Reynolds number, where inertia terms have little or no effect upon the flow of fluid. There is a vast variety of non- Newtonian fluids. The examples of such substances in the chemical laboratories and industries include polymers, synthetic lattices, protein solutions, special soap solution etc. and those encountered in daily life include asphalts, paints, pinch, starch suspensions, marine glue and certain honeys. With the growing importance of non–Newtonian fluids in modern technology and industries, the investigations of such fluids are desirable. Fredricksen (1964) has given a good review of non–Newtonian fluids whereas Joseph (1976) has also considered the stability of viscous-elastic fluids. There are many visco-elastic fluids which cannot be characterized either by Maxwell's constitutive relations or by Oldroyd's constitutive relations. One of such visco-elastic fluids are Walters' (model B μ) fluid. Walters (1960) has proposed a constitutive equation for such type of elastic–viscous fluids. Many other research workers have paid their attention towards the study of Walters' (model B μ) fluid. The mixture of polymethyl methacrylate and pyridine at 25.0 grams containing 30.5 grams of polymers per litre behaves very nearly as the Walters' (model B μ) visco-elastic fluid and which is proposed by Walters (1962). This class of fluids is used in the manufacture of parts of spacecrafts, aero plane, tires, belt conveyors, ropes, cushions, seats, foams, plastics, engineering equipments etc.

When a fluid slowly percolates through the pores of an isotropic and homogeneous porous medium, the gross effect is represented by Darcy's law which states that usual viscous term in the equations of Walters' (model B μ) fluid motion is replaced by the resistance term

$$-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)U$$

Where μ and μ' are the viscosity and the visco-elasticity of the Walters' (model B μ) fluid, k_1 is the medium permeability and U is the Darcian (filter) velocity of the fluid. The flow through porous medium has been of considerable interest in recent years, particularly in geophysical fluid dynamics, recovery of crude oil from the pores of reservoir rocks. Also the flow through porous media is of interest in chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil physics, biophysics. When we consider flow in a porous medium, some additional complexities arise which are principally due to the interactions between the fluid and the porous material. Here, we will consider those fluids for which Darcy's law is applicable. The stability of the flow of a single component fluid through a porous medium taking into account the Darcy's resistance has been studied by Lapwood (1948) and Wooding (1960). The effect of the Earth's magnetic field on the stability of such a flow is of interest in geophysics particularly in the study of Earth's core where the Earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of differential diffusion. The physical properties of comets and meteorites strongly suggest the importance of porosity in astrophysical context [McDonnel (1978)].

Sharma and Kango (1999) have studied the stability of two superposed Walters' (model B μ) visco-elastic fluids in the presence of suspended particles and variable magnetic field in porous media. Sharma and Kumar (2001) have studied the Rayleigh–Taylor instability of stratified Walters' (model B μ) in the presence of suspended particles and variable horizontal magnetic field and have found that the criteria determining stability are independent of the effects of viscosity and visco-elasticity. The magnetic field stabilizes the system. The visco-elasticity of the medium has damping effects on the growth rates but has enhanced effects for certain ranges of the wave-numbers. Sharma and Kumar (2000) have studied magneto-gravitational instability of a thermally conducting rotating Walters' (model B μ) fluid with Hall current. Goren and Gottlieb (1982) have studied the surface-tension-driven breakup of visco-elastic liquid threads. The instability of plane interface between two uniformly superposed and streaming fluids through a porous medium has been studied by Sharma and Spanos (1982). Keeping in mind the various applications mentioned above, our interest, in the present paper, is to deal with the instability of streaming visco-elastic Walters' (model B μ) fluids in a pMagneticsium in hydro-magnetics.

2. Formulation Of The Problem And Perturbation Equations

The initial state whose stability we wish to examine is that of an incompressible, electrically conducting Walters' (model B μ) elastico - viscous fluid in which there is a horizontal streaming in the x - direction with a velocity $\vec{U}(z)$ through a homogeneous and isotropic porous medium. A uniform horizontal magnetic field pervades the system. The character of the equilibrium of this initial state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let $\rho, \upsilon, \upsilon', \vec{U}(U(z),0,0)$ and $\vec{H}(H(z),0,0)$ denote respectively the density, the kinematic viscosity, the kinematic visco-elasticity, the pressure, the velocity of fluid and the magnetic field. Let g, k_1 and stand for the acceleration due to gravity, medium permeability and medium porosity respectively. Suppose that at some prescribed levels z , density may change discontinuously and bring into play effects due to effective interfacial tensions T and let $\vec{\eta}_s$ denote the normal to the macroscopic interface. Then the equations of motion, continuity, incompressibility and Maxwell's equations for the elastico - viscous Walters' (model B μ) fluid through a porous medium are given by

$$\frac{\rho}{\varepsilon}\left(\frac{\partial \vec{U}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{U}.\nabla\right)\vec{U}\right)=-\nabla p+\left[T_s\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)z_s\right]\delta(z-z_s)\vec{n}_s$$

$$-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)\vec{U}+\frac{1}{4\pi}(\nabla\times\vec{H})-\rho g\vec{\lambda}, \tag{1}$$

$$\nabla.\vec{U}=0,\quad \nabla.\vec{H}=0 \tag{2}$$

$$\varepsilon\frac{\partial p}{\partial t}+\left(\vec{U}.\nabla\right)\rho=0, \tag{3}$$

$$\varepsilon\frac{\partial \vec{H}}{\partial t}=\nabla\times\left(\vec{U}\times\vec{H}\right), \tag{4}$$

Then the linearized perturbation equations become

$$\frac{\rho}{\varepsilon}\left(\frac{\partial u_1}{\partial t}+\frac{U}{\varepsilon}\frac{\partial u_1}{\partial x}+\frac{u_3}{\varepsilon}\frac{\partial U}{\partial z}\right)=-\frac{\partial}{\partial x}\delta p-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)u_1, \tag{5}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial u_2}{\partial t}+\frac{U}{\varepsilon}\frac{\partial u_2}{\partial x}\right)=-\frac{\partial}{\partial y}\delta p-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)u_2+\frac{H}{4\pi}\left(\frac{\partial h_y}{\partial x}-\frac{\partial h_x}{\partial y}\right), \tag{6}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial u_3}{\partial t}+\frac{U}{\varepsilon}\frac{\partial u_3}{\partial x}\right)=-\frac{\partial}{\partial z}\delta p-\frac{\rho}{k_1}\left(\nu-\nu'\frac{\partial}{\partial t}\right)u_3$$

$$+\frac{H}{4\pi}\left(\frac{\partial h_z}{\partial x}-\frac{\partial h_x}{\partial z}\right)+T_s\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\delta z_s\right]\delta(z-z_s)-g\delta\rho, \tag{7}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}+\frac{\partial u_3}{\partial z}=0, \tag{8}$$

$$\left(\varepsilon\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\delta\rho=-u_3\frac{d\rho}{dz}, \tag{9}$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0, \tag{10}$$

$$\left(\varepsilon\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right)\vec{h}=H\frac{\partial}{\partial x}\vec{u}\;+h_z D\vec{U}, \tag{11}$$

In equation (7), δz_s can be expressed in terms of the normal component of the velocity and since

$$\left(\varepsilon\frac{\partial}{\partial t}+U_s\frac{\partial}{\partial x}\right)\delta z_x = u_{3_s}, \tag{12}$$

where a subscript 's' distinguishes the value of the quantity at z = z_s Analysing the disturbances into normal modes, we seek solutions whose dependence on x, y and t is of the form

$$\exp i(k_x x + k_y y + \eta t). \tag{13}$$

Where η is the growth rate, $k=\sqrt{k_x^2+k_y^2}$ is the resultant wave number and k_x, k_y are the horizontal wave numbers. Eliminating and u_1, u_2 and δp from equations (5) - (7) and using (8) - (12) and expression (13), we obtain

$$D\left[\left\{\frac{i\rho}{\varepsilon}\left(\eta+\frac{k_xU}{\varepsilon}\right)+\frac{\rho}{k_1}(v-inv')\right\}Du_3-\frac{ik_x\rho}{\varepsilon^2}(DU)u_3\right]$$

$$+\frac{ik^2{}_xH^2k^2}{4\pi(\varepsilon n+k_xU)}u_3-k^2\left\{\frac{i\rho}{\varepsilon}\left(n+\frac{k_xU}{\varepsilon}\right)+\frac{\rho}{k_1}(v-inv')\right\}u_3$$

$$-igk^2\left[(D\rho)-\frac{k^2T_s}{g}\delta(z-z_s)\right]\frac{u_3}{\varepsilon n+k_xU}-ik_x\frac{H^2}{4\pi}D\left[\frac{1}{(\varepsilon n+k_xU)}\right.$$

$$\left.\left\{k_xDu_3-\frac{k^2(DU)}{\varepsilon n+k_xU}u_3-\frac{ik_y^2(DU)\left(\frac{\rho}{\varepsilon^2}-\frac{k_x^2H^2}{4\pi(\varepsilon n+k_xU)^2}\right)u_3}{\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(v-iv'n)-\frac{ik_x^2H^2}{4\pi(\varepsilon n+k_xU)}}\right\}\right]$$

$$-\frac{ik_xk_y^2H^2}{4\pi}D\left[\frac{\frac{\rho}{k_1}(v-iv'n)(DU)u_3}{\frac{i\rho}{\varepsilon^2}(\varepsilon n+k_xU)+\frac{\rho}{k_1}(v-iv'n)-\frac{ik_x^2H^2}{4\pi(\varepsilon n+k_xU)}}\right]=0, \tag{14}$$

Two Uniform Streaming Fluids Seperated By A Horizontal Boundary: Let two uniform Walters'(model B μ) fluids of densities ρ_1 and ρ_2 be separated by a horizontal boundary at z = 0 and the density ρ_2 of the upper fluid be less than the density ρ_1 of the lower fluid so that, in the absence of streaming, the configuration is a stable one and the porous medium throughout is assumed to be isotropic and homogeneous. Let the two fluids be streaming with velocities U_1 and U_2 . Then in each region of constant $\rho, \upsilon, \upsilon'$ and U, equation (14) reduces to

$$(D^2 - k^2)u_3 = 0 \tag{15}$$

The boundary conditions to be satisfied are:

(i) Since U is discontinuous at z = z_s, the uniqueness of the normal displacement of any point on the interface implies, according to (12), that

$$\frac{u_3}{\varepsilon n + k_x U} \tag{16}$$

Must be continuous at the interface.

(ii) Integrating (14) between $z_s - \eta$ and passing to the limit $\eta = 0$, we obtain, in view of (16), the jump condition

$$\Delta_s\left[\left\{\frac{i\rho}{\varepsilon}\left(\eta + \frac{k_x U}{\varepsilon}\right) + \frac{\rho}{k_1}(v - iv'\eta)\right\}Du_3 - \frac{ik_x^2 H^2}{4\pi}\Delta_s\left\{\frac{Du_3}{\varepsilon\eta + k_x U}\right\}\right]$$

$$= igk^2\left[\Delta_s(\rho) - \frac{k^2 T_s}{g}\right]\left(\frac{u_3}{\varepsilon\eta + k_x U}\right)_s, (for\, z = z_s) \tag{17}$$

While the equation valid everywhere else is $(z \neq z_s)$ is

$$D\left[\left\{\frac{i\rho}{\varepsilon}\left(\eta + \frac{k_x U}{\varepsilon}\right) + \frac{\rho}{k_1}(v - iv'\eta)\right\}Du_3\right] - k^2\left\{\frac{i\rho}{\varepsilon}\left(\eta + \frac{k_x U}{\varepsilon}\right)\right.$$

$$\left. + \frac{\rho}{k_1}(v - iv'\eta)\right\}u_3 - \frac{ik_x^2 H^2}{4\pi(\varepsilon\eta + k_x U)}(D^2 - k^2)u_3$$

$$= igk^2(D\rho)\left(\frac{u_3}{\varepsilon\eta + k_x U}\right). \tag{18}$$

Here $\Delta_s(f) = f(z_s + 0) - f(z_s - 0)$ is the jump which a quantitative experience at the interface $z = z_s$ and the subscript 's' distinguishes the value, a quantity known to be continuous at an interface, takes at $z = z_s$ Since $\dfrac{u_3}{\varepsilon\eta + k_x U}$ must be continuous on the surface z = 0 and cannot increase exponentially on either side of the surface, the solutions appropriate for the two regions are u_3 .

$$u_{3_1} = A(\varepsilon\eta + k_x U_1)e^{+kz}, \quad (z < 0) \tag{19}$$

$$u_{3_2} = A(\varepsilon\eta + k_x U_2)e^{-kz}, \quad (z > 0) \tag{20}$$

Applying the boundary conditions (17) to the solution (19) and (20), we obtain the dispersion relation

$$\left[1 - \frac{\varepsilon}{k_1}(\alpha_1 v'_1 + \alpha_2 v'_2)\right]\eta^2 + \left[\frac{2k_x}{\varepsilon}(\alpha_1 U_1 + \alpha_2 U_2) - \frac{i\varepsilon}{k_1}(\alpha_1 v_1 + \alpha_2 v_2) - \frac{k_x}{k_1}(\alpha_1 v'_1 U_1 + \alpha_2 v'_2 U_2)\right]\eta$$

$$+\left[\frac{k_x^2}{\varepsilon^2}\left(\alpha_1 U_1^2+\alpha_2 U_2^2\right)-\frac{ik_x}{k_1}\left(\alpha_1 v_1 U_1+\alpha_2 v_2 U_2\right)-2k_x^2 V_A^2-gk\left\{\left(\alpha_1-\alpha_2\right)+\frac{k^2 T_s}{g\left(\rho_1+\rho_2\right)}\right\}\right]=0 \qquad (21)$$

Where $v_1(=\mu_1/\rho_1)$, $v_2(=\mu_2/\rho_2)$ and $v'_1(=\mu'_1/\rho_1)$, $v'_2(=\mu'_2/\rho_2)$
are the kinematic viscosities and kinematic viscoelasticities of fluids 1 and 2 respectively.

$$\alpha_1=\frac{\rho_1}{\rho_1+\rho_2},\quad \alpha_2=\frac{\rho_2}{\rho_1+\rho_2},\quad \text{and}\quad V_A^2=\frac{H^2}{4\pi\left(\rho_1+\rho_2\right)}$$

is the square of Alfven velocity. Consider now the special case in which the lower and upper fluids are streaming with velocities U(=U_1) and -U(=U_2) respectively. Equation (21) reduces to

$$\left[1-\frac{\varepsilon}{k_1}\left(\alpha_1 v'_1+\alpha_2 v'_2\right)\right]\eta^2+\left[\frac{2k_x}{\varepsilon}(\alpha_1-\alpha_2)U-\frac{i\varepsilon}{k_1}(\alpha_1 v_1+\alpha_2 v_2)\right.$$

$$\left.-\frac{k_x}{k_1}(\alpha_1 v'_1-\alpha_2 v'_2)U\right]\eta+\left[\frac{k_x^2 U^2(\alpha_1+\alpha_2)}{\varepsilon}-\frac{ik_x}{k_1}(\alpha_1 v_1-\alpha_2 v_2)U\right.$$

$$\left.-2k_x^2 V_A^2-gk\left\{\left(\alpha_1-\alpha_2\right)+\frac{k^2 T_s}{g\left(\rho_1+\rho_2\right)}\right\}\right]=0. \qquad (22)$$

Equation (22) yields

$$i\eta=\frac{1}{1-\frac{\varepsilon}{k_1}\left(\alpha_1 v'_1+\alpha_2 v'_2\right)}\left[\left\{-\frac{\varepsilon}{2k_1}\left(\alpha_1 v_1+\alpha_2 v_2\right)-\frac{ik_x}{\varepsilon}\left(\alpha_1-\alpha_2\right)U+\frac{ik_x}{2k_1}\left(\alpha_1 v'_1-\alpha_2 v'_2\right)U\right\}\right.$$

$$\pm\left\{\frac{\varepsilon}{4k_1^2}\left(\alpha_1 v_1+\alpha_2 v_2\right)^2+k_x^2 U^2\left(\frac{4\alpha_1\alpha_2}{\varepsilon^2}-\frac{1}{16k_1^2}\left(\alpha_1 v'_1-\alpha_2 v'_2\right)^2+\frac{1}{4\varepsilon k_1}\left(\alpha_1-\alpha_2\right)\left(\alpha_1 v'_1-\alpha_2 v'_2\right)\right)\right.$$

$$\left.\left.-\frac{2ik_x\alpha_1\alpha_2 U}{k_1}\left(v_1-v_2\right)-2k_x^2 V_A^2-gk\left(\left(\alpha_1-\alpha_2\right)+\frac{k^2 T_s}{g\left(\rho_1+\rho_2\right)}\right)\right\}^{1/2}\right] \qquad (23)$$

We now discuss several cases of interest.

(a) Absence of surface tension (Ts = 0): In the absence of the surface tension equation (23) becomes

$$i\eta = \frac{1}{1-\frac{\varepsilon}{k_1}(\alpha_1 \nu'_1 + \alpha_2 \nu'_2)}\Bigg[\left\{-\frac{ik_x}{\varepsilon}(\alpha_1-\alpha_2)U - \frac{\varepsilon}{2k_1}(\alpha_1\nu_1+\alpha_2\nu_2) + \frac{ik_x}{2k_1}(\alpha_1\nu'_1-\alpha_2\nu'_2)U\right\}$$

$$\pm\left\{\frac{\varepsilon}{4k_1^2}(\alpha_1\nu_1+\alpha_2\nu_2)^2 + k_x^2U^2\left(\frac{4\alpha_1\alpha_2}{\varepsilon^2} - \frac{1}{16k_1^2}(\alpha_1\nu'_1-\alpha_2\nu'_2)^2 + \frac{1}{4\varepsilon k_1}(\alpha_1-\alpha_2)(\alpha_1\nu'_1-\alpha_2\nu'_2)\right)\right.$$

$$\left. -\frac{2ik_x\alpha_1\alpha_2U}{k_1}(\nu_1-\nu_2) - 2k_x^2V_A^2 - gk(\alpha_1-\alpha_2)\right\}^{1/2}\Bigg] \tag{24}$$

When k_x = 0, equation (24) gives:

$$i\eta = \frac{1}{1-\frac{\varepsilon}{k_1}(\alpha_1 \nu'_1 + \alpha_2 \nu'_2)}\Bigg[\left\{-\frac{\varepsilon}{2k_1}(\alpha_1\nu_1+\alpha_2\nu_2)\right\}$$

$$\pm\left\{\frac{\varepsilon^2}{4k_1^2}(\alpha_1\nu_1+\alpha_2\nu_2)^2 + gk(\alpha_2-\alpha_1)\right\}^{1/2}\Bigg] \tag{25}$$

and the kinematic viscoelasticities υ'_1, υ'_2 of the two fluids are assumed to be the same ($\upsilon'_1 = \upsilon'_2 = \upsilon'$) then one of the values of *in* is positive, which means that perturbations grow with time and so the system is unstable. When $\alpha_2 > \alpha_1$ and $\upsilon' > \frac{k_1}{(\alpha_1+\alpha_2)\varepsilon}$ then one of the values of *in* is negative, which means that perturbations decay with time and so the system is therefore stable.

When $\alpha_1 > \alpha_2$ and $\upsilon' < \frac{k_1}{(\alpha_1+\alpha_2)\varepsilon}$ and the kinematic visco-elasticities υ'_1, υ'_2 of the two fluids are assumed to be the same ($\upsilon'_1 = \upsilon'_2 = \upsilon'$), then both the values of *in* are either real, negative or complex conjugates with negative real parts. The system is therefore stable. Equation (25) also shows that perturbations transverse to the direction of streaming $(k_y \neq 0)$.

are unaffected by the presence of streaming for the special case when perturbations in the direction of streaming are ignored $(k_x = 0)$. (ii) In every other direction, instability occurs when

$$k_x^2U^2\left[\frac{4\alpha_1\alpha_2}{\varepsilon^2} - \frac{\nu'}{4k_1}(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1} - \frac{1}{\varepsilon}\right)\right] > gk(\alpha_1-\alpha_2) + 2k_x^2V_A^2, \tag{26}$$

i.e., for a given difference in the velocity 2U and are for a given direction of the wave vector k, instability occurs for all wave numbers,

$$k > \frac{g(\alpha_1-\alpha_2)}{\left[\frac{4\alpha_1\alpha_2 U^2}{\varepsilon^2} - \frac{\nu'}{4k_1}U^2(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1}-\frac{1}{\varepsilon}\right) - 2V_A^2\right]\cos^2\theta}, \tag{27}$$

$$k_{\min} = \frac{g(\alpha_1-\alpha_2)}{\left[\frac{4\alpha_1\alpha_2}{\varepsilon^2} - \frac{\nu'}{4k_1}(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1}-\frac{1}{\varepsilon}\right)\right]U^2 - 2V_A^2} \tag{28}$$

which the system is unstable. The instability of the system is thus postponed. We thus obtain the stabilizing effect of the magnetic field and kinematic viscoelasticity.

(b) Presence of Surface Tension: When surface tension is present, equation (23) yields stability if

$$\left[\frac{4\alpha_1\alpha_2}{\varepsilon^2} - \frac{\nu'}{4k_1}(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1}-\frac{1}{\varepsilon}\right)\right]U^2 - 2V_A^2 < g\left\{\frac{(\alpha_1-\alpha_2)}{k} + \frac{kT_s}{g(\rho_1+\rho_2)}\right\}, \tag{29}$$

$$\frac{g(\alpha_1-\alpha_2)}{k} = \frac{kT_s}{\rho_1+\rho_2} \tag{30}$$

If k* denotes the value of k given by (30), we have stability

$$\left[\frac{4\alpha_1\alpha_2}{\varepsilon^2} - \frac{\nu'}{4k_1}(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1}-\frac{1}{\varepsilon}\right)\right]U^2 - 2V_A^2 < \frac{2T_s k^*}{(\rho_1+\rho_2)} \tag{31}$$

Substituting the value of k* in accordance with (30), we obtain

$$\left[4 - \frac{\varepsilon^2\nu'}{4k_1\alpha_1\alpha_2}(\alpha_1-\alpha_2)^2\left(\frac{\nu'}{4k_1}-\frac{1}{\varepsilon}\right)\right]U^2 < \frac{2\varepsilon^2}{\alpha_1\alpha_2}\sqrt{\frac{T_s g(\alpha_1-\alpha_2)}{\rho_1+\rho_2}} + \frac{2\varepsilon^2}{\alpha_1\alpha_2}V_A^2. \tag{32}$$

For non - porous medium and in the absence of kinematic visco-elasticity and magnetic field (H= 0), the equation (32) yields the result as due to Kelvin [cf. Chandrasekhar (1961), inequality (40), p. 486], implying thereby that the magnetic field and surface tension have stabilizing effects and completely suppresses the Kelvin - Helmholtz instability for small wavelength perturbations. The medium porosity and viscoelasticity reduce the stability range given in terms of a difference in streaming velocities and the Alfven velocity.)

Interfacial Tension Term In Porous Media: In flows through porous media, there are no sharp fronts and so no actual interfacial tensions at some prescribed levels zs, as in ordinary fluid dynamics. However, there is a macroscopic interface (broad front), if viewed from a large distance, and in analogy with Laplace's formula, at each point of the macroscopic interface

$$(p_1 - p_2)_{z=zs} = -T_s(c_1 + c_2),$$

Where is an "effective interfacial tension" and are the signed principal curvatures of the macroscopic interface. This is the first approximation to the problems as in actual practice there is no "effective interfacial tension" but in the absence of any better theory, this is being used as suggested by Chouke et al (1959). s T , 1c 2 c

Let $\delta z_s(x,y,t)$ denote the perturbation in surface of separation z_s and η_s stands for the normal to the interface. In the equation (8), use has been made of the approximations)

$$c_1 \sim -\frac{\partial^2}{\partial x^2}(\delta z_s), \quad c_2 \sim -\frac{\partial^2}{\partial y^2}(\delta z_s),$$

Since the study of viscoelastic fluids is, in many situations desirable in geophysics, chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil physics, biophysics and pulp technology, therefore a study has been made of the instability of streaming Walters'(model B μ) viscoelastic fluids in a porous medium in the presence of a magnetic field.

3. Conclusion

In this analysis we have shown that the stability of an interface between two Walters' (model B μ) viscoelastic fluids in a porous medium in the presence of a magnetic field is affected by the streaming of the fluids in a direction parallel to the interface. An illustration of this phenomenon is obtained by considering a less dense fluid overlying a more dense fluid in a porous medium in the presence of a magnetic field, the interface between the two fluids will remain stable for all perturbations of all wavelengths. However when flow parallel to the interface was included in the analysis, the interface was found to be unsuitable for sufficiently small wavelengths. Further, on including surface tension, magnetic field and visco-elasticity, these instabilities were found to be stabilized unless the difference in velocities exceeded a critical value given by equation (32). Thus for perturbations having large wavelengths in comparison with those determined by equation (28), the stabilizing effects of the density difference dominate. Also equation (31) illustrates that perturbations, which have smaller wavelengths, are stabilized by the surface tension, magnetic field and the visco-elasticity. The formation of instabilities is, therefore, restricted to the cases where these two effects do not overlap. Thus, the medium porosity and the kinematic viscoelasticity reduce the stability range given in terms of a difference in streaming velocities and Alfven velocity.

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